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Machine learning: The valuation edge
A new approach to estimating real-time NAVs in private equity secondaries

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Introduction

This paper illustrates how private equity (PE) investors may gain an edge in secondary transactions by utilizing machine learning (ML) techniques. In particular, we introduce Pantheon’s “NAV now-casting” model and discuss how it can be a helpful reference point when pricing secondary deals. While the underwriting of secondary transactions is ultimately driven by the quantitative and qualitative due diligence led by investment team professionals, the ever-increasing availability of data in private markets has brought new opportunities to gain an information edge that can help to identify value creation opportunities – at least for those investors that have access to the information from GPs, and the data engineering and ML capabilities to harness and interpret it at scale. We showcase Pantheon’s NAV now-casting tool as a tangible example of one of the many ways that ML can be integrated in the investment process.

Opportunities and challenges in PE secondary markets

At their core, secondary investments seek to generate strong risk-adjusted returns by identifying investment opportunities with significant embedded value and growth opportunities. This can be achieved by acquiring assets that can generate large capital gains relative to holding NAV, by buying the assets at a discount to the holding NAV, or both.

At first sight, buying assets at a discount to the most recently available NAV may appear to be a more reliable way to acquire assets with embedded value, as it does not require taking a view on the development of future NAVs. However, things are not that straightforward: NAVs for private assets are typically released with a significant time lag, so the latest and most relevant NAVs are generally unknown at the time of bidding. For instance, investors bidding on a deal on 31st March 2021 would ideally like to buy at discount to Q1 2021 NAV, but will typically just know, and be expected to bid off, Q4 2020 valuations – or in some cases even earlier accounts. Movements in valuation will impact the relative discount (or premium) paid vs the NAV: as Exhibit 1 shows, a hypothetical pre-
mium to the latest available marks can actually equate to an “effective discount” – i.e. a discount to the NAV prevailing at the time of the transaction bid date.

Post-account date movements in NAV can have a material impact on the effective discount. The typical reporting delay by GPs ranges from 2 to 4 months. Over these time spans, changes in valuations can be substantial, especially in highly volatile periods. This is illustrated in Exhibit 2, which reports quarterly changes in fund returns (see box), for a sample of U.S. buyout (US BO) funds\(^2\). The typical (median) fund tends to report quarterly returns between 0% and 5%, with a few exceptions: Q4 2010 (+7%), Q3 2011 (-3%), and during the Covid pandemic (-8% in Q1 2020, followed by 8%, 9%, and 9% in Q2-Q4). However, there is a lot of dispersion around the median fund – 15%, on average, when looking at the difference between top and bottom decile returns.

**Jargon buster**

**Fund returns** are calculated by dividing the quarterly change in NAVs, after taking into account movements in cash flows (fund calls and distributions), by the previously published NAV. This is equivalent to estimating valuation changes, so we use the terms interchangeably throughout the paper. Please see the appendix for further details.

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**Exhibit 1: Post-account valuation movements and effective discounts**

For illustration purposes. The diagram assumes that no calls or distributions occur in between quarters.

**Exhibit 2: Quarterly returns of U.S. buyout funds**

How may NAV now-casting help?

To overcome the pricing challenge described in the previous section, we would ideally like a tool that estimates NAVs and effective discounts in real time, prior to the release of official figures by GPs. This is exactly what a “now-casting” model aims to accomplish. For instance, a NAV now-casting model could generate an estimate of Q1 2021 NAV on 31st March – potentially months ahead of GP reports. Note that, true to its name, the model estimates NAV values for the current and past, rather than future, quarters; the model does not fore-cast, it now-casts. The idea is simple: leverage published data from readily available reference points such as public indices, and feed it into a model based on historical correlations with fund valuations to produce an estimate for the current quarter.

Historically, now-casting has been commonly used by macroeconomists to get current estimates of low frequency, lagged macro-variables such as GDP. Notably, policy makers such as Central Banks use these models as a real-time gauge of the economy. Early models have been built based on traditional econometric and statistical approaches, but recent efforts have also explored the potential of ML. While the focus of most studies remains on the estimation of macroeconomic variables, the scope of applications has widened and spilled over into the world of PE thanks to the work of Brown et al (2020).

Building on this literature, our work explores applications of this technology to PE secondary markets. In particular, we envisage two use cases of NAV nowcasts in secondary deals:

1. To estimate the effective discount of a proposed bid
2. To calculate the bid level required to achieve the desired effective discount

A first stab at the problem: naive now-casting models

To warm up to the challenge of developing a NAV now-casting model, it is useful to start by examining the “roll for cash” (R4C) approach. R4C updates NAVs only based on call and distribution activity, effectively assuming no change in valuations. Despite its apparent naïveté, the approach is widely adopted as it is easy to implement: the calculation is straightforward, the model does not require any “training”, and its inputs are fund-level calls and distributions, which are readily-available in real time to PE investors. As such, it can act as a robust benchmark for more sophisticated NAV now-casting models.

Exhibit 3 illustrates the performance of R4C as a NAV now-casting tool in our sample. For each quarter, we

Exhibit 3: Errors of R4C model

Median, 10th percentile, and 90th percentile of the return absolute errors produced by the R4C model. Source: Pantheon analysis of U.S. Buyout funds in Pantheon database, accessed April 2021. See Appendix for details on methodology and data.
show the distribution of absolute errors – the degree to which returns based on the NAVs published later by GPs differed from the R4C estimate – across all fund observations for that quarter. The median error fluctuates around 5% of reported NAV, but the performance of R4C is highly dependent on the quarter: in volatile quarters such as Q1 and Q2 2020, for instance, the median error was 8% and 10%, respectively. However, these median values say little about the performance of R4C across the full range of funds, especially about the half of the universe where model errors are in excess of the 5% median: the 10th percentile of errors hovers around 13%. This indicates that high single- and low double-digit errors are not uncommon for R4C, which can have material implications for secondary pricing.

An obvious enhancement to R4C is to adjust for changes in public markets. After all, intuition suggests that we should be able to do a better job than R4C by exploiting the correlation between private and public markets. We call this the “roll for publics” (R4P) model and use the S&P 500 as an adjustment factor. Exhibit 4 shows the performance of the model in our sample, in terms of its improvement vs R4C. While there are quarters where the R4P offers a slight improvement vs R4C, there are also quarters where it significantly underperforms. Overall, R4P underperforms R4C by 11%, on average.

To get a sense of why the R4P does not improve upon the R4C, the top panel of Exhibit 5 compares the distribution of fund returns in different public market regimes. For R4P to be a good model of PE returns, we would need to observe a “beta” of close to 1, which would denote that the average return of PE moves in line with the public market. Instead, we notice that median PE returns fluctuate less markedly than market returns, and that such dampening effect is “non-linear”: it is less accentuated for large market swings. Moreover, as shown in the left vs right panels of Exhibit 5, PE returns appear to be driven in part by market returns in the previous quarter, which is something that the R4P model does not take into account: this pattern is also non-linear, and in fact resembles a U-shape. Finally, there is a large degree of dispersion across PE funds, especially during periods in which market swings are larger. Some of these GP valuation patterns may depend on fund-level characteristics such as age, which are ignored by the R4P model.

Exhibit 4: improvement of R4P vs R4C

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Most of these observations are inherent properties of GP valuations. For our purposes, it would make sense to allow our model to learn these patterns of the data and improve estimates upon the R4C and R4P benchmarks. We turn to this task in the next section.

Leveraging the power of ML

How, then, can we empower our NAV now-casting model to harness the patterns in the data to produce better estimates than the naive R4C and R4P models?

Traditionally, the challenge has been tackled by following the classic statistical / econometric recipe:
1. Posit a model (typically linear) of the data
2. Estimate the model by running “regressions” on the full sample of data to quantify the impact on a set of known variables
3. Run a battery of tests to assess the “statistical significance” of the model structure and explanatory variables
4. Repeat steps 1-3 until the model “cannot be rejected” by the data

More recently, ML has offered an alternative approach that is more model-agnostic and data-driven:
1. Define a range of alternative models, including richer, non-linear models
2. Estimate all models in parallel using most, but not all, sample data – the “training” set
3. Use a second portion of the data – the “validation” set – to fine tune the models, and select the winning one that produces the smallest degree of error
4. Use the third and final portion of the data – the “testing” set – to get an unbiased estimate of how well the chosen model can generalize in unseen data

The power of ML over traditional statistics lies in the richness, and number, of models it explores, and in the potential to handle high dimensional data and non-linear effects. But this richness also creates the
risk of overfitting the data, i.e. producing a model that mistakes noise for signal and, as a consequence, does not generalize well on unseen data. Hence, much of the practice of ML seeks to strike a balanced trade-off between learning from data whilst retaining the ability to generalize beyond it. The complex and non-linear behaviours in GP valuations – as illustrated in Exhibit 5 – are potentially fertile ground for ML. At the same time, it is important to manage the risk of overfitting, especially with noisy and relatively "small" financial datasets.

At Pantheon, we decided to tackle the challenge of NAV now-casting by embracing the ML approach. In 2019, we developed and deployed a model prototype focused on U.S. buyouts. In particular, we trained and validated the performance of over 10,000 ML models, and eventually chose a lead candidate that we deployed in Q3 2019. All of the models considered make use of fund-level information and ignore portfolio company data. This not only enables a like-for-like comparison of our model with R4C and R4P, which do not use company data either, but also provides a tool for situations where company data is not available. Since knowledge of company data can only enhance the precision of the models, performance of our model can be viewed as a lower bound for what can be achieved with richer information sets. The appendix explains the detail of our data and methodology, including how we balanced the opportunity to explore a wide and complex set of models with the risk of selecting a model that would over-fit sample data and fail to generalize in a production environment.

**Our approach in practice**

With the benefit of six quarters of live performance, we are now able to look back to how well our ML-inspired approach has delivered against its potential. Immediately following the end of each quarter, we generated now-casts for a sample of U.S. BO funds; as we received actual NAV figures from GPs, we then calculated the error of our now-casts. Exhibit 6 shows the performance of our model up to Q4 2020, the latest

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**Exhibit 6: Improvement of Pantheon model vs R4C and R4P**

<table>
<thead>
<tr>
<th>Year</th>
<th>% Improvement vs R4C</th>
<th>% Improvement vs R4P - S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td></td>
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<td>2016</td>
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<td>2020</td>
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</table>

quarter for which we have actual NAV data, in terms of improvements vs our R4C and R4P benchmarks. To provide a longer back-test of the approach, we also report the performance of the model between Q1 2017 and Q2 2019, which we used as a “test set” when we originally developed the algorithm (see technical appendix for more details).

The data shows that, with the exception of just a single quarter in the case of R4C (Q4 2018) and three quarters in the case of R4P (Q2 2016, Q3 2020 and Q4 2020), our model improved the accuracy of NAV nowcasts consistently and, often, considerably – in some cases by as much as 50%. We consider this to be a strong vindication of our ML approach.

Conclusion

This paper provides a practical illustration of how estimating GP valuations in real time may help the secondary investment process and how ML can be harnessed to develop NAV now-casting models. We have shown how a NAV now-casting tool can improve the accuracy of GP valuations estimates, and therefore be a helpful reference point when assessing secondary transactions. In particular, the NAV now-caster can be valuable for top-down screening of secondary opportunities – ahead of engaging in detailed bottom-up due diligence.

With this work, we are only scratching the surface of the potential uses of data and ML-powered applications in private markets. We believe that this area of research can offer great opportunities to identify value creation opportunities for those investors that have access to data and that possess the quantitative capabilities to interpret such data effectively.
Deep-dive break-out boxes

Traditional stats vs ML

Traditional statistical / econometric methods and machine learning have a lot in common. For starters, both approaches aim to find patterns in data that can be generalized – with particular focus on the prediction of discrete and continuous outcomes, otherwise known as “classification” and “regression” problems. Given the similarities in goals, overlap in methods, and privileged role that data plays in both disciplines, it is sometimes hard to understand where the boundaries lie. For instance – is classic linear regression a statistical or ML tool?

We believe that the ultimate driver of the distinction between the two fields is about the nature of the assumptions that an analyst is willing to make. Statistical / econometric methods tend to be “model rich”: they make probabilistic assumptions about the data generating process, and use these rich assumptions to develop estimators and statistical tests that can be used to separate the wheat (“signal”) from the chaff (“noise”). For instance, a statistician using standard linear regression will be happy to claim that the OLS estimator delivers the most probable estimates, is the most “efficient” estimator, and will also be ready to ignore estimates that are not deemed “statistically significant” on the basis of tests. How can the statistician be so confident? Simple: all these claims are a mathematical consequence of the model assumptions that the statistician is willing to accept.

ML methods, on the other hand, tend to be “model-agnostic”, “data-rich”, and laser-focused on the problem of prediction. No assumptions are made about the data generating process, and any model will do, as long as its predictions generalize well out of sample. This pragmatic goal is easier said than done: data-driven models have the bad habit of overfitting sample data, sometimes reaching the extreme point of perfectly mimicking sample noise, and therefore often fail to work on unseen data. The generalization problem is a challenge for statistical approaches as well, but is particularly acute for ML precisely because of the few assumptions it makes about the data generating process: there is no “theory” to guide the choice of the model and distinguish signals from noise. ML deals with the generalization issue in the standard data-driven way, by carefully carving out sample data (“validation” and “testing” sets) to simulate out-of-sample performance, and employing “regularization” techniques to alleviate overfitting concerns. Turning back to the linear regression example, an ML analyst would judge the model based on its simulated predictive performance, pruning it based on regularization and validation techniques, without any concern around how close the model is to a theory of where the data come from, unless such concern is instrumental to making a better prediction.

In short, a simple regression can be seen both as statistical and ML tool – based on the philosophy underlying the approach, and the practical differences in model R&D approach.
In recent years there had been a lot of enthusiasm around the ability of ML to crack a variety of challenges – including image and speech recognition, language translation, and problems in life sciences such as protein folding and drug discovery. Three ingredients have typically contributed to the success of ML in these applications: little or no change in data patterns over time and samples; abundance of data; and rich model architectures such as “deep learning” models. With these ingredients, the recipe has been straightforward: create models that can fit the data very well, safe in the knowledge that there is enough data to train the model on and that the model is unlikely to fail in a live setting.

Unfortunately, the same ingredients are not typically available in financial data, which can be “small” and subject to the adaptive nature of evolving markets. As a consequence, the recipe of “throwing large models at the data” is unlikely to work either: there is just not enough data to train large models and, even if there were, large models would be likely to pick up ephemeral in-sample noise rather than signal, and therefore fail at performing well out of sample\(^9\).

The idiosyncrasies of financial data, however, do not mean that there is no role for ML to play in finance: rather, ML should be cognizant of these limitations and craft approaches that are appropriate for the problem at hand. In particular, as argued by several researchers\(^10\), applications of ML to financial problems should strive to minimize the risk of overfitting via combination of strategies, including: focusing on models with parsimonious architectures; applying regularization techniques; and conducting rigorous validation. When the right balance is struck, ML can indeed be of great value add: Gu et al (2018), for instance, show that it can generate better stock return predictions than traditional forecasting methods, largely due to their ability to capture nonlinear effects that are missed by traditional statistical and econometric techniques.

### Technical appendix

**Description of the NAV now-casting problem**

We denote NAVs, calls, and distributions by \(N\), \(C\), and \(D\), respectively, and use subscripts to indicate the fund \(f\) and quarter \(q\) they refer to. Cash flows and valuations over consecutive quarters are linked by the standard accounting identity:

\[
N_{f,q} = N_{f,q-1}(1 + r_{f,q}) + C_{f,q} - D_{f,q}
\]

where \(r_q\) is the quarterly uplift, or return, on opening NAV. The equation implicitly defines quarterly returns as:

\[
r_{f,q} = \frac{(N_{f,q} - N_{f,q-1} - C_{f,q} + D_{f,q})}{N_{f,q-1}}
\]
While calls and distributions are readily observable as they happen, NAVs are published by GPs only after a reporting lag, so $N_{f,q}$ is usually not known until some weeks, or months, after $q$. As a consequence, at the end of quarter $q$, $N_{f,q}$ can be viewed as a random variable, and the purpose of a NAV now-casting tool is precisely to estimate this quantity. Since $C_{f,q}$, $D_{f,q}$, and $N_{f,q-1}$ can be assumed to be known at quarter $q$, estimating $N_{f,q}$ is equivalent to estimating $r_{f,q}$, so the “NAV now-casting” problem is equivalent to the “return now-casting” problem. For convenience we use the return now-casting representation in the paper.

Sample data

To construct the sample, we start off with 209 U.S. buyout funds from 2000 – 2020 vintages in Pantheon’s dataset, and discard all fund-quarter observations prior to 2010. We drop observations where fund age is either less than 1 year, or greater than 12 years. As a result, we obtain a sample of 2,724 fund-quarter observations, which we split into a “training set” (all data between 2010:Q1 and 2014:Q4), “validation set” (between 2015:Q1 and 2016:Q4), and “test set” (between 2017:Q1 and 2020:Q4).

Sample data includes quarterly observations on fund returns and a set of fund-level features observable at the end of each quarter:

- Market returns: current quarter and 3 lags
- Previous quarterly uplifts: 3 lags
- Fund age
- GP fixed effects
- Quarter fixed effects
- Fund cash flows: current quarter

Hypothesis space

Let $h_\theta(x_{f,q})$ be the “hypothesis”, or model, that predicts returns based on the feature vector $x$ and parameter vector $\theta$. The objective of the ML problem is to find a hypothesis that approximates observed returns as close as possible, in a mean absolute error sense. We use two simple hypotheses as benchmarks:

- Roll for cash (R4C): $h_\theta(x_{f,q}) = 0$
- Roll for publics (R4P): $h_\theta(x_{f,q}) = m_q$, where $m_q$ is the return on a public equity benchmark (S&P500 in our analysis).

To construct the space of hypotheses we explore, we consider different combinations of algorithms, features, and regularization techniques for a total of 10,000 models. We define a model as a combination of input features, algorithm type, and regularization technique. The range of algorithms considered encompass the usual suspects - OLS regressions, elastic nets, PLS, SVM, neural networks – and accompanying regularization techniques.

Model training, validation, and selection

We split the sample into a training, validation, and testing set while preserving the chronological order of the data. That is: first, we estimate models using data from 2010 to 2015; next, we validate the models and select the winner with data from 2015 to 2017; finally, we gauge the performance of the winning model with test data from 2017 to Q2 2019, which is augmented with actual model performance from Q3 2019 to Q4 2020. The design minimizes the risk of overfitting, as discussed above, and removes any forward looking bias: the performance of the model in the test data between 2017-2020 reflects the actual out-of-sample accuracy of the model.

After estimating the models in the training set, we use the validation set for model tuning and selection. We take a decision-theoretic approach and select models based on a CRRA utility criterion evaluated over the quarterly outperformance of the model vs R4C in the validation set. Using this approach allows us to select a model that not only outperforms R4C on average between 2015-2017, but also, thanks to the concavity of the CRRA utility criterion, does it reliably. Pantheon’s NAV now-casting tool is an ensemble of the three models that maximize the utility criterion.
Endnotes

1 Other rationales for including secondary investments in a portfolio may include mitigating the J-curves of private equity programs, generating short-term liquidity, increasing the speed of deployment of a PE program, calibrating portfolio construction, consolidating or expanding GP relationships, etc.

2 See Appendix for details on methodology and data.


4 See Banbura et al (2010) and (2013) for an overview of now-casting techniques, and Andreou et al (2011) for the “mixed data sampling” (MIDAS) method.

5 See Babii et al (2020).

6 Absolute error: absolute difference between actual return and return predicted by the model. Since the R4C effectively assumes 0 returns, note that the absolute return error generated by the R4C model for a specific fund/quarter observation is equal to the absolute value of the return of the fund in that quarter. See technical appendix for further details.

7 We also assessed the R4P model using the Russell 2000, a common proxy of U.S. small and mid caps, and found that the performance was worse than using the S&P500.

8 Typically within the first 5 business days of the new quarter.

9 For a in-depth discussion of this topic, see AQR’s Alternative Thinking Report (2019).


11 By 2010 most funds should have adopted the mark-to-market rules prescribed by FAS 157.

References


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